Exceedance Statistics of Accelerations Resulting from Thruster Firings on the Apollo-Soyuz Mission

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Introduction

THE acceleration environment of spacecraft and rockets has been a subject of concern to designers[1,2] and experimenters[3-7] alike. Spacecraft acceleration resulting from firings of Vernier control system thrusters is an important consideration in the design, planning, execution, and postflight analysis of laboratory experiments in space. In particular, scientists and technologists involved with the development of experiments to be performed in space in many instances require statistical information on the magnitude and rate of occurrence of spacecraft accelerations. Typically, these accelerations are stochastic in nature, so that it is useful to characterize these accelerations in statistical terms. This paper summarizes statistics of spacecraft accelerations associated with thruster firings that occurred during the Apollo-Soyuz mission. The motivation for performing this statistical analysis was simply to obtain an idea of the kind of exceedance statistics one might expect for vehicle accelerations associated with Vernier control system thruster firings. The statistics of spacecraft accelerations for any given space mission depend on charactreistics of the spacecraft (mass, control systems, etc.), mission profile, and a host of other parameters unique to the mission.

Thruster Acceleration Time Histories

Acceleration at time t at a given point in a spacecraft as a result of thruster firings is given by[8]

$$g(t) = \dot{\boldsymbol{\Omega}} \times \boldsymbol{r} + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \boldsymbol{r}) + \boldsymbol{a} \tag{1}$$

where $\boldsymbol{\Omega}$ is the rigid body angular velocity vector of the spacecraft, $\boldsymbol{r}$ is the position vector relative to the spacecraft center of mass, $\boldsymbol{a}$ is rectilinear acceleration, and $(\dot{\ })$ denotes time differentiation. During a thruster firing event, $\dot{\boldsymbol{\Omega}}$ and $\boldsymbol{a}$ are nonzero. After the thruster event, $\boldsymbol{a}$ vanishes (assuming no other forces are acting on the vehicle) and $|\dot{\boldsymbol{\Omega}}| \sim |\boldsymbol{\Omega}|^2$. Furthermore, the second term on the right side of Eq. (1) between thruster events is usually two to three orders of magnitude smaller than the remaining terms during a thruster event (at least for the Apollo-Soyuz mission and typical Space Shuttle missions). In addition, the typical thruster event duration time for attitude Vernier control is less than a second. Thus, a time history of the magnitude of spacecraft acceleration, at a given point in the spacecraft, as a result of Vernier control system thruster firings would be like a series of Dirac delta function-like acceleration spikes, corresponding to the thruster firings, with essentially zero acceleration between the spikes. Of course, the total time history would have additional contributions from crew activity, venting of gases and liquids, atmospheric drag, etc. This Note does not consider these additional contributions.

Time histories of the kind just mentioned were calculated for specific periods of time during the Apollo-Soyuz mission for postflight analysis of the experiments performed on that mission. The calculations involved the estimation of $\boldsymbol{\Omega}$ and $\boldsymbol{a}$ via the calculation of the net force and net torque associated with each thruster firing event. A discussion of the Apollo-Soyuz Vernier control system and thrusters is provided in Ref. 9, and the data are given in Ref. 10.

Four acceleration time histories were selected for analysis, namely, those associated with experiments MA-044, -085, -041, and the science demonstation (Sci-Dem). These runs were approximately 3 to 17 h in duration. The thruster firing time for each thruster firing event was 0.015 or 0.031 s.

Statistical Calculations

We have performed statistical calculations on the individual components and the magnitude of $\boldsymbol{g}$. In this section we present our statistical calculations on the magnitude of $\boldsymbol{g}$. We seek the answers to two questions. First, what is the average rate of exceedance of vehicle acceleration at some assigned level g_l? Second, what is the statistical distribution of the time interval between adjacent acceleration events?

Acceleration Exceedance Statistics

We define the exceedance rate N of the magnitude of the acceleration vector, g to be the average number of acceleration events that exceed level g_l per unit time. To calculate this quantity from the data we merely count the number of events that exceed level g_l. In an attempt to combine the exceedance calculation for the four time histories that were examined we 1) scaled the exceedance rate N with the standard deviation, σ_T, of the time interval between acceleration events and 2) subtracted the mean value of g, i.e. $\bar{g}$, from g and scaled the resulting difference with the standard deviation of g, i.e. σ_g. Thus Fig. 1 provides plots of results of calculations of nondimensional exceedance rate $N\sigma_T$ as a function of the nondimensional unbiased accelerations, $(g-\bar{g})/\sigma_g$.

Distribution of Time Intervals

To calculate the distribution of time intervals between acceleration events, T, straightforward counting procedures were used. Here again, in an attempt to combine the results of our calcualtions, we subtracted the mean time interval, $\bar{T}$, from the time interval and scaled the resulting difference with σ_T. Thus Fig. 2 provides plots of the distribution function of unbiased time interval between acceleration events, $(T-\bar{T})/\sigma_T$.

Discussion

It appears that some order can be brought about in spacecraft Vernier control thruster firing statistics via the scaling introduced in the previous section. This is especially true for the thruster time interval distribution function. The thruster acceleration exceedances do not collapse as well as the time interval distribution functions for $(g-\bar{g})/\sigma_g < -1$. However, this may not be significant because the important part of the curve is for $(g-\bar{g})/\sigma_g > 0$; i.e., the part for $g > \bar{g}$. The part associated with $(g-\bar{g})/\sigma_g < 0$ encompasses $0 \le g \le \bar{g}$.

We do not claim these results to be universal because of variation in mass, crew activity, control system parameters, etc., from one spacecraft to the next. If results like these *could be derived for planned missions, they could play a useful role in the planning of experiments which are sensitive to spacecraft dynamics. These statistics permit the calculation of the risk associated with* $g \ge g_c$ for a given experiment duration time t_e, where g_c is a critical value of g to the experiment. To do this, one needs to assume that the exceedances of the g process above level g_c arrive independently. We note by $Q(t_e)$ the number of exceedances of g at level g_c over the experiment duration time t_e. Clearly, the process $Q(t_e)$ is a Poisson

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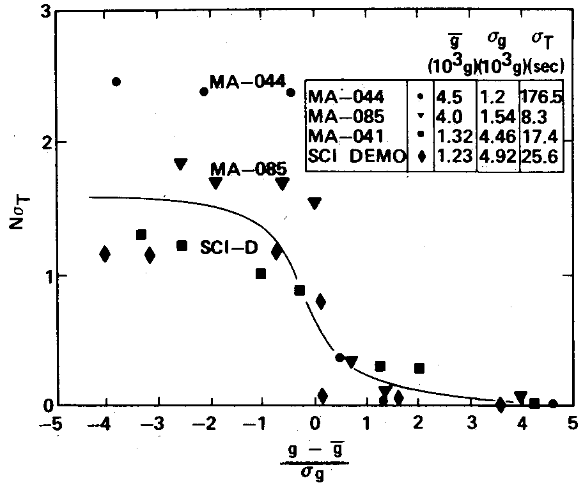

Fig. 1 Apollo-Soyuz exceedance statistics of accelerations associated with Vernier control system thruster firings.

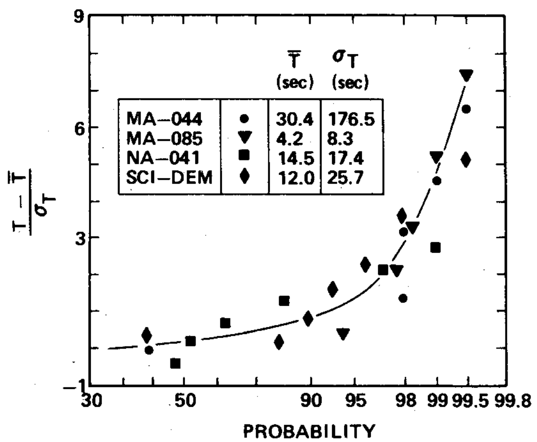

Fig. 2 Distribution of time intervals between Vernier control system thruster firings for Apollo-Soyuz mission.

process, and the probability of $Q(t_e)$ being less than or equal to an assigned value (for example, q) is given by Ref. 11:

$$P[Q(t_e) \leq q, t_e] = \frac{(\lambda t_e)^q}{q!} e^{-\lambda t_e} \tag{2}$$

where λ is a parameter. The probability of no exceedance of the g process above the critical value g_c in time interval t_e follows by setting $q=0$ in the preceding equation, so that

$$P[Q(t_e) = 0, t_e] = e^{-\lambda t_e} \tag{3}$$

By definition of the Poisson process we set $\lambda = N$. Now the risk R that the g process will exceed the critical value of g_c at least once during an experiment of duration t_e is thus

$$R = 1 - e^{-Nt_e} \tag{4}$$

Upon specification of risk R and t_e the value of N can be calculated. A curve like that shown in Fig. 1 can be used to calculate g upon specification of σ_T, $\bar{g}$, and σ_g. These latter parameters, as well as the $N\sigma_T$ vs $(g-\bar{g})/\sigma_g$ curve, will in all likelihood vary from mission to mission of a given spacecraft and possibly vary within a mission. A curve like that shown in Fig. 2 can be used to estimate the distribution of time intervals between g events upon specification of $\bar{t}$ and σ_t. This distribution would provide an estimate of extreme and typical values of t. Thus one obtains a statistical description of the dynamic environment in terms of the risk of exceeding a critical g level and the distribution of time intervals between g events.

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